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Heat transfer in the boundary layer of a 'gas-evaporating drops' two-phase mixture

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Abstract—A mist flow in a laminar boundary layer over a hot flat plate is studied by asymptotic and numerical methods. A two-continuum approximation is used to describe the 'gas-evaporating drops' two-phase medium. The Saffman force, acting on the drops and causing their deposition, is taken into account. The mechanism of heat transfer enhancement by the drops evaporating in the boundary layer has been studied, and similarity criteria have been found. The predicted overall heat transfer coefficient for a plate segment is in good agreement with the familiar experimental data.

1. INTRODUCTION

THE CURRENT interest in the study of the laws governing the thermal interaction of two-phase flows with solid surfaces is dictated by a wide range of problems encountered in technical applications. Primarily, these are the problems of predicting the optimal thermal regimes of thermal power engineering equipment with a two-phase working body (heat exchangers, steam generators, combustion chambers, etc.). It has been determined experimentally that the presence of even a small amount of a dispersed admixture can lead to substantial changes in heat fluxes to solid surfaces immersed in two-phase flows [1, 2]. Owing to the great diversity in the two-phase wall flows, the construction of a consistent theory with allowance for the inertial properties of particles (drops) has been only partly developed. Closed mathematical models have been proposed only for several typical, but the most simple, flows: a laminar boundary layer in dusty gas on a plate, in the inlet lengths of a channel and a tube, in the stagnation point vicinity and a number of others [3–8]. The wall flows of 'gas drops' mixtures require even more complicated models which would take into consideration the possible formation of a liquid film on a surface immersed in flow [9] and also phase transitions on the surfaces of drops (evaporation). The present work deals with the construction of the theory for a laminar boundary layer of a 'gas-drops' mixture on a hot flat surface in axial flow under the conditions when two-velocity and two-temperature effects, and also drop evaporation, are significant. Previously, evaporation of single drops in a boundary layer has been investigated in ref. [10]. Within the framework of a one-velocity model, the boundary layer in a vapour-drop medium has been analysed in ref. [11] where, in particular, the possibility of the formation of a pure vapour wall layer has been shown.

Below, a two-continuum approximation is adopted, and a consistent asymptotic model of a 'gas-drops' mixture boundary layer is constructed. By taking as an example the axial flow past a hot flat plate, a number of limiting situations are studied (low concentration of drops, absence of drop deposition, drop deposition is substantial), and numerical and asymptotic solutions of the constructed boundary layer equations have been found. The main result is the identification of the similarity numbers and explanation of the mechanism underlying a sharp increase in heat transfer of a surface immersed in flow in the presence of a small amount of evaporating drops in the boundary layer. For the regime of inertial deposition of drops, an agreement was found between the predicted overall heat transfer coefficient of the plate and experimental data [2]. In this paper, the model of a two-phase boundary layer on surfaces in axial flows [3–5, 7] has been extended to the case of the presence of phase transitions on particle surfaces.

2. BASIC EQUATIONS AND STATEMENT OF THE PROBLEM

Consider the usual assumptions of the model of a dust-laden gas with a negligibly small volume concentration of the particles on the surfaces of which a phase transition is taking place [12, 13]. The drops are considered to be monodisperse, with time-dependent (due to evaporation) radius σ and mass m. The drop substance vapour is the carrying phase. Besides the Klyachko form of the aerodynamic drag force [14], the expression for the interphase momentum exchange will involve the Saffman lift force [15], the importance of which grows rapidly with the size of the particle moving in the boundary layer [8]. Then, in the Cartesian coordinate system, the force acting on the drop, will acquire the form

NOMENCLATURE

a	dimensionless rate of drop evaporation
b	dimensionless coefficient in equation (9)
c_1, c_2, c_3	auxiliary functions in equation (19)
c_p, c_s	specific heats of vapour and drops
Ec	Eckert number
e, g, r, w	auxiliary functions in equations (18) and (19)
f	force acting on a drop
$\mathbf{f}$	unit vector along the x-axis
F	Blasius function
G, D	correction factors allowing for inertia effects in drag force theory and heat transfer laws
H	heat of vaporization
J	drop evaporation rate
k	dimensionless coefficient
l	length of drop retardation
L	length of plate segment
m	mass of a drop
n_s	number density of drops
Pr	Prandtl number
q	heat flux
R	maximum Reynolds number of flow around drops
Re	Reynolds number
T	temperature
u, v	velocity components
$\mathbf{v}$	velocity
x, y	coordinates
X, Y	stretched coordinates.

Greek symbols

α	mass concentration of drops
ε	small parameter
ζ	self-similar variable of boundary layer
η	boundary layer variable
κ	dimensionless parameter characterizing the Saffman force magnitude
μ, ν	dynamic and kinematic viscosities
ρ	density
σ	drop radius
χ, φ	dimensionless parameters characterizing the gain in heat transfer.

Subscripts

s	dispersed phase parameters
∞	external flow parameters
0	initial value
1, 2, 3	subscripts of new variables.

Superscript

*	dimensional quantities.

$$\mathbf{f}_s = 6\pi\sigma^*\mu^*(\mathbf{v}^*-\mathbf{v}_s^*)(1+\tfrac{1}{6}Re_s^{2/3}) + 6.46\sigma^{*2}\left(\mu^*\rho^*\frac{\partial u^*}{\partial y^*}\right)^{1/2}(u^*-u_s^*)\mathbf{j}. \quad (1)$$

Here and hereafter, the asterisk denotes the dimensional quantities, the subscript s refers to the parameters of the medium of particles, $\mathbf{j}$ signifies the unit vector along the y^*-axis, $Re_s = 2\sigma^*\rho^*|\mathbf{v}^*-\mathbf{v}_s^*|/\mu^*$. The expression for the heat flux to a drop will be adopted in the form [13]

$$q_s^* = 4\pi\sigma^*\lambda^*(T^*-T_s^*)(1+0.3Re_s^{1/2}\,Pr^{1/3}). \quad (2)$$

Here λ^* is the thermal conductivity of the carrying phase, $Pr = c_p\mu^*/\lambda^*$, c_p is the specific heat of the carrying phase at constant pressure. Note that in expressions (1) and (2) the effects associated with the evaporation of particles are not taken into account. These effects require negligibly small corrections in equations (1) and (2) [12] in the considered case of a weak injection on the particle surface. The transfer coefficients are assumed to be constant. The evaporation process is considered to be equilibrium. The drop surface temperature is related to the ambient vapour pressure by the Clapeyron–Clausius condition. Then, while moving in a boundary layer at a constant pressure, the drop will have a constant temperature which will be equal to the temperature of equilibrium evaporation. The mass flux from the drop surface J^* per unit time will be defined in this case from the condition

$$J^*H = q_s^*.$$

Here H is the latent heat of vaporization. Now, the equations for a vapour-drop mixture will be written under the adopted assumptions for flow in the boundary layer of a hot semi-infinite plate having the prescribed temperature T_w^*. The origin of the Cartesian coordinate system x^*, y^* will be fixed on the plate tip. To non-dimensionalize, the relaxation length of phase velocities with the Stokes resistance law [4] will be taken as the longitudinal length scale. The dimensionless variables will be introduced as follows:

$$x = \frac{x^*}{l}, \quad \eta = \frac{y^*}{l\sqrt{\varepsilon}}, \quad u = \frac{u^*}{u_\infty^*}, \quad u_s = \frac{u_s^*}{u_\infty^*}$$

$$v_s = \frac{v_s^*}{u_\infty^*\sqrt{\varepsilon}}, \quad v = \frac{v^*}{u_\infty^*\sqrt{\varepsilon}}, \quad \sigma = \frac{\sigma^*}{\sigma_\infty^*}$$

$$n_s = \frac{n_s^*}{n_{s\infty}^*}, \quad T = \frac{T^*-T_w^*}{T_\infty^*-T_w^*}, \quad T_s = \frac{T_s^*-T_w^*}{T_\infty^*-T_w^*}$$

$$l = \frac{m_\infty^* u_\infty^*}{6\pi\sigma_\infty^*\mu^*}, \quad \varepsilon = \frac{\mu^*}{\rho^* u_\infty^* l}.$$

Here n_s is the number density of drops, $1/\varepsilon$ is the

Reynolds number based on the relaxation length of phase velocities, the subscript ∞ refers to the external flow parameters.

When $\varepsilon \ll 1$ in the boundary layer approximation [4], the equations of the vapour-drop mixture [12] for the case of an incompressible carrying phase will acquire the following form. The continuity equations of phases are

$$\frac{\partial(n_s u_s)}{\partial x}+\frac{\partial(n_s v_s)}{\partial \eta}=0, \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial \eta}=\alpha a J n_s. \quad (3)$$

The notations are introduced $J=\sigma(1-T)D$

$$\alpha=\frac{\rho^*_{s\infty}}{\rho^*}, \quad a=\frac{2c_p(T^*_w-T^*_\infty)}{3H\,Pr}$$

$$D=(1+0.3Pr^{1/3}\,R^{1/2}|u-u_s|^{1/2}).$$

The momentum equations of phases will be

$$\sigma^2\left(u_s\frac{\partial u_s}{\partial x}+v_s\frac{\partial u_s}{\partial \eta}\right)=f_x$$

$$\sigma^2\left(u_s\frac{\partial v_s}{\partial x}+v_s\frac{\partial v_s}{\partial \eta}\right)=(v-v_s)G+\kappa\sigma(u-u_s)\sqrt{\left(\frac{\partial u}{\partial \eta}\right)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial \eta}+\alpha n_s\sigma f_x+\alpha a n_s(u-u_s)J=\frac{\partial^2 u}{\partial \eta^2}. \quad (4)$$

Here

$$f_x=(u-u_s)G, \quad G=1+\tfrac{1}{6}R^{2/3}|u-u_s|^{2/3}$$

$$R=\frac{2\sigma^*_\infty u^*_\infty\rho^*}{\mu^*}, \quad \kappa=\frac{6.46}{12\pi\sqrt{6}}R^{3/2}\left(\frac{2\rho_s^0}{\rho^*}\right)^{1/4}$$

where ρ_s^0 is the drop substance density. The drop evaporation equation is

$$u_s\frac{\partial \sigma}{\partial x}+v_s\frac{\partial \sigma}{\partial \eta}=-\frac{aJ}{3\sigma^2}. \quad (5)$$

The heat influx equation is given as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial \eta}=\frac{1}{Pr}\frac{\partial^2 T}{\partial \eta^2}-Ec\left(\frac{\partial u}{\partial \eta}\right)^2$$

$$+\alpha\,Ec\,n_s\sigma f_x(u-u_s)+\alpha a J n_s(1-T)+\frac{2\alpha}{3\,Pr}n_s J. \quad (6)$$

Here

$$Ec=\frac{u^{*2}_\infty}{c_p(T^*_w-T^*_\infty)}.$$

In the external flow, the velocity and phase equilibrium is implied. Then the boundary conditions will become

$$\eta=0: \quad u=v=T=0, \quad \eta\to\infty: \quad u=T=1$$

$$x=0: \quad u_s=T_s=\sigma=n_s=1, \quad v_s=0. \quad (7)$$

The formulated system of equations for a vapour-drop boundary layer depends on five similarity parameters, introduced above: the Prandtl number Pr, the Eckert number Ec, the relative mass drop concentration α, the parameter which characterizes the drop evaporation rate a, and finally, the parameter characterizing the contribution of the Saffman force to the interphase momentum exchange κ.

Table 1

σ^*_∞ (cm)	l (cm)	R	κ
10^{-4}	0.014	1.33	0.69
10^{-3}	1.44	13.3	22.7
10^{-2}	144	133	717.5

It should be noted that the adopted approximation of the incompressible carrying phase is applicable only at small Mach numbers of the external flow and small temperature differences in the boundary layer. For simplicity, the transfer coefficients of the carrying phase are assumed to be constant.

3. SIMPLIFICATION OF THE MODEL FOR CHARACTERISTIC LIMITING CASES

The system of equations (3)–(7) will be simplified by neglecting the terms which are small for the usual conditions of mist flow origination and movement. The magnitude of the governing parameters will be evaluated for the case, when $u^*_\infty=10^3$ cm s^{-1}, $\rho_s/\rho^{0*}=10^3$, $\rho^*=0.001$ g cm^{-3}, $v^*=0.15$ cm^2 s^{-1}, $H=2300$ J g^{-1}, $T^*_w-T^*_\infty=50$°C, $c_p=2$ J deg g^{-1} and for three sizes of drops (see Table 1). In this case, the Eckert number and the scale of the drop evaporation rate are $Ec=10^{-3}$, $a=0.043$. It is clear from Table 1 that the parameter κ, i.e. the role of the Saffman force, grows quickly with the drop size. For large drops, the Saffman force is decisive for the process of their deposition and heat transfer with the surface immersed in flow. The smallness of parameters Ec and a and also of α (which usually does not exceed several per cent) allows one to discard in equations (3)–(7) the terms of the order of Ec and of the product αa. Then, the boundary layer equations will acquire the form

$$\frac{\partial(n_s u_s)}{\partial x}+\frac{\partial(n_s v_s)}{\partial \eta}=0$$

$$u_s\frac{\partial u_s}{\partial x}+v_s\frac{\partial u_s}{\partial \eta}=\frac{(u-u_s)G}{\sigma^2}$$

$$u_s\frac{\partial v_s}{\partial x}+v_s\frac{\partial v_s}{\partial \eta}=\frac{(v-v_s)G}{\sigma^2}+\frac{\kappa(u-u_s)}{\sigma}\sqrt{\left(\frac{\partial u}{\partial \eta}\right)}$$

$$u_s\frac{\partial \sigma}{\partial x}+v_s\frac{\partial \sigma}{\partial \eta}=\frac{a(T-1)D}{3\sigma}, \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial \eta}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial \eta}+\alpha\sigma n_s(u-u_s)G=\frac{\partial^2 u}{\partial \eta^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial \eta}=\frac{1}{Pr}\frac{\partial^2 T}{\partial \eta^2}+\frac{2\alpha}{3\,Pr}n_s\sigma(1-T)D. \quad (8)$$

The system of equations (8) makes it possible to analyse the significant effect of evaporating drops on the process of heat exchange with the surface exposed to flow. Since the Saffman force expels the drops onto the wall [8], the overall heat flux to the surface is equal to the sum of heat fluxes due to the heat conduction of the carrying phase q_1^* and owing to the contact heat transfer and evaporation of depositing drops q_2^*. The presence of drops can lead to an increase in q_1^*, as compared with q_0^* (the heat flux to the wall) in the case of the absence of drops.

Let us investigate the solution of equations (8) for two limiting cases:

(a) the deposition of drops is insignificant and heat transfer is governed by the vapour thermal conductivity $q_1^* \gg q_2^*$; and

(b) the evaporation of depositing drops is the basic mechanism of heat transfer enhancement $q_2^* \gg q_1^* \sim q_0^*$.

4. THE ABSENCE OF THE DEPOSITION OF DROPS

Consider the case of small drops, when the Saffman force may be neglected. It is assumed in equations (8) that $G = D = 1$ and $\kappa = 0$. As experience in numerical calculations shows [4], the effect of the concentration of particles on the carrying phase velocity field can be neglected, if the particle concentration is low ($\alpha \leqslant 0.1$). This is explained by the smallness of the source term $\alpha n_s(u-u_s)$ in the carrying phase momentum equation even in the region of the accumulation of particles. In the case under consideration, $\sigma \leqslant 1$, therefore, the indicated estimate for the source term in the gas momentum equation will still be valid. Hence, in the approximation assumed, the effect of drops is described only by the source term in the heat influx equation for the carrying phase. In the region of the accumulation of drops this effect can be substantial.

As follows from ref. [4], at $\sigma = 1$, a non-integrable singularity of particle concentration [16] appears on the plate surface and this leads to the necessity of adding to the complexity of the particles medium model. It will be shown that in the case of evaporating drops the concentration singularity is integrable and originates only at one point of the retardation of drops. Behind this point, a region of pure steam is being developed in the boundary layer. At the edge of this region $\sigma = 0$.

In order to study the asymptotic structure of flow near the plate surface and near the stagnation point of drops, introduce a formal small parameter η_0. This is the value of the ordinate prior to which the longitudinal velocity profile of the carrying phase at the stagnation point of drops can be considered as linear with a prescribed accuracy, i.e. $u = k\eta$, where k is a constant. Then with $\eta \leqslant \eta_0$ there will be the following value orders: $u \sim \eta_0$, $v \sim \eta_0^2$. The value orders of the dispersed phase parameters on the length scale $x \sim 1$, $\eta \sim \delta\ (\eta_0)$ (where $\delta \to 0$ when $\eta_0 \to 0$) are as follows

$$u_s \sim 1, \quad \sigma \sim 1, \quad v_s \sim \delta^2.$$

In this region the equations for main terms of the expansion of solution (8) are of the form

$$u_s \frac{\partial u_s}{\partial x} = -\frac{1}{\sigma^2}, \quad u_s \frac{\partial \sigma}{\partial x} = -\frac{a}{3\sigma}, \quad \frac{\partial (n_s u_s)}{\partial x} = 0.$$

The solution of these equations is as follows

$$u_s = (1-bx)^{1/b}, \quad \sigma = (1-bx)^{a/3b}$$

$$n_s = \frac{1}{u_s}, \quad b = \frac{2a+3}{3}. \tag{9}$$

Now, the asymptotic solution near the stagnation point of drops will be found. Introduce a new coordinate system and stretched variables

$$x_1 = x - \frac{1}{b}, \quad \eta_1 = \frac{\eta}{\delta}, \quad X = \frac{x_1}{\eta^b}, \quad Y = \frac{\eta}{\eta_0}.$$

In this region, the dispersed phase parameters will be sought in the form:

$$\eta_0^{a/3}\sigma_0(X, Y) + \cdots, \quad \eta_0 u_{s0}(X, Y) + \cdots$$

$$\frac{1}{\eta_0} n_{s0}(X, Y) + \cdots.$$

Then, the equations for the main expansion terms will acquire the form

$$u_{s0}\frac{\partial \sigma_0}{\partial X} = -\frac{a}{3\sigma_0}, \quad u_{s0}\frac{\partial u_{s0}}{\partial X} = \frac{kY - u_{s0}}{\sigma_0^2}, \quad \frac{\partial n_{s0}u_{s0}}{\partial X} = 0. \tag{10}$$

The conditions of the matching with solution (9) on the scale $x \sim 1$ for $X \to \infty$ yield

$$\sigma_0 \sim (-bX)^{a/3b}, \quad u_{s0} \sim (-bX)^{1/b}, \quad n_{s0}u_{s0} \sim 1.$$

Solution (10) can be presented in the form

$$\frac{\sigma_0^b}{b} + \frac{kY\sigma_0^2}{2} = -\frac{aX}{3} + B(X, Y)$$

$$u_{s0} = kY + \sigma_0^{3/a}, \quad n_{s0} = \frac{1}{u_{s0}}. \tag{11}$$

The condition of the matching with the solution in the region $x \sim 1$ gives

$$B(X, Y) = \frac{kY}{2}(bX)^{2a/3}. \tag{12}$$

Now, the equation will be found for the surface on which drops evaporate completely. Equations (11) and (12) yield that $\sigma = 0$ on the surface the equation of which in the initial coordinate has the form

$$\eta = \frac{2a}{3k}\left(\frac{1}{b}\right)^{2a/3b}\left(x - \frac{1}{b}\right)^{1/b}. \tag{13}$$

It is seen that the concentration of drops tends to infinity only at one point on the plate surface at

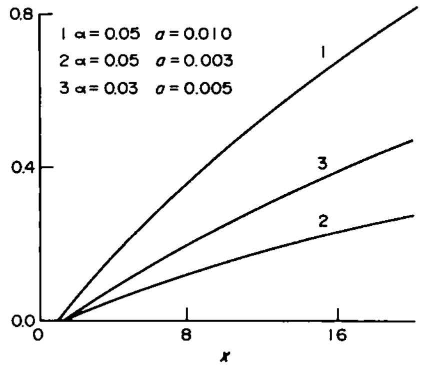

FIG. 1. The dependence of the thickness of a pure vapour layer on a and α.

$x = 1/b$. The concentration singularity is integrable, consequently, the used model of non-interacting particles remains applicable [16].

To quantitatively determine the effect exerted by drops on heat flux to the wall due to their accumulation near the surface (13), numerical calculations of equations (8) were conducted at $\kappa = 0$, $G = D = 1$ and neglecting the effect of drops on the carrying phase field velocity. For numerical calculations, system (8) was rewritten in new variables x, $\zeta = \eta/\sqrt{x}$, and new unknown functions $v_1 = \sqrt{x}v$, $v_{s1} = \sqrt{x}v_s$, $g = n_s u_s$ were introduced. The finite-difference method [4] was employed. To the right of the stagnation point of drops, the lower boundary conditions for the difference equations referring to the dispersed phase were fulfilled on the surface, equation (13).

The results of calculations of the boundary of the pure vapour region are presented in Fig. 1. Curves 1 and 2 correspond to the values $\alpha = 0.05$; $a = 0.01$ and 0.003; curve 3 corresponds to $a = 0.005$ and $\alpha = 0.03$. It is evident that within the boundary layer the relative thickness of the pure vapour region grows quickly. In Fig. 2 the carrying phase temperature profile is

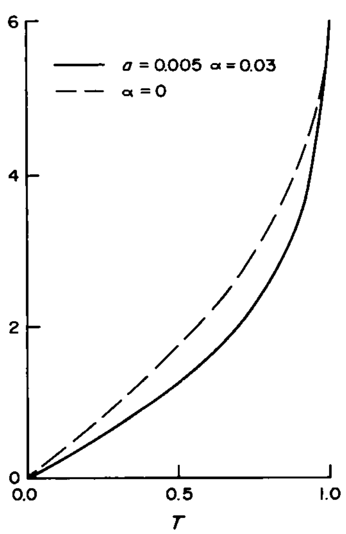

FIG. 2. The profile of the carrying phase temperature, formed far from the plate tip.

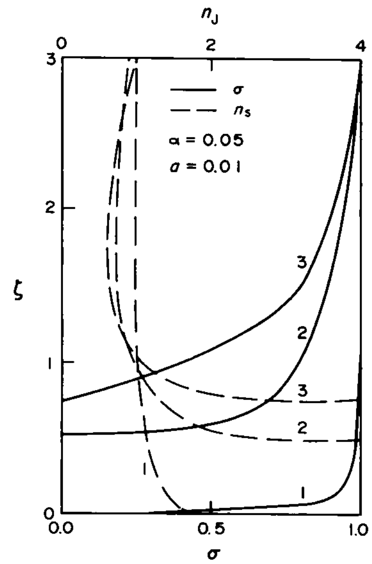

FIG. 3. Development of profiles of concentration and radii of drops along the longitudinal coordinate. Curves 1 correspond to $x = 0.5$ for $n_s(\zeta)$ and $x = 0.993$ for $\sigma(\zeta)$, curves 2 correspond to $x = 11$, curves 3 to $x = 25$.

presented which is calculated for $\alpha = 0.005$ and $a = 0.01$ and which is formed far from the leading edge. When $x \gg 11$, this profile becomes virtually self-similar. The dashed line presents the temperature profile in a pure gas with which $T(\zeta)$ coincides at small values of x. Figure 3 demonstrates the development of the dispersed phase parameters $\sigma(\zeta)$ (solid lines) and $n_s(\zeta)$ (dashed lines) along the longitudinal coordinate at $\alpha = 0.05$ and $a = 0.01$. Curve 1 corresponds to $x = 0.5$ for n_s and $x = 0.993$ (the stagnation point) for σ; curves 2 and 3 correspond to the values $x = 11$ and 25, respectively. Note that n_s reaches the finite value at the boundary of the pure vapour region everywhere except the drop stagnation point. Calculation results for the relative increase in the heat flux Nu/Nu_0 on the plate are presented in Fig. 4 (Nu_0 is the Nusselt number in the case of pure vapour). Curves 1–3 were calculated for the values $a = 0.003$; $\alpha = 0.11, 0.05$ and 0.03, respectively; and curve 4 for the values $a = 0.01$ and $\alpha = 0.03$. It is seen that the ratio Nu/Nu_0 virtually ceases to depend on x with the increase in the distance from the leading edge. The results of systematic calculations show that this limiting value for different small values of a and α depends only on the ratio

$$\chi = \frac{\alpha}{a} = \frac{3\alpha H\, Pr}{2c_p(T_w^* - T_\infty^*)}. \tag{14}$$

Curve 5 in Fig. 4 represents the dependence (at large values of x) of the quantity Nu/Nu_0 on χ. Thus, in the case considered the parameter χ is the only similarity parameter which describes the degree of heat transfer enhancement for linear scales greatly exceeding the relaxation length of the velocities of phases. In this case, the heat transfer enhancement effect is explained by the accumulation and simul-

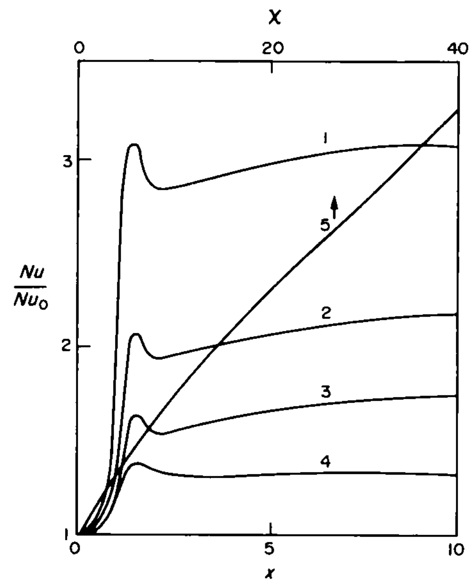

FIG. 4. Local heat transfer coefficient vs a and α (curves 1–4) and its limiting value ($x \gg 1$) vs χ (curve 5) (1, $a = 0.003$, $\alpha = 0.1$; 2, $a = 0.003$, $\alpha = 0.05$; 3, $a = 0.003$, $\alpha = 0.03$; 4, $a = 0.01$, $\alpha = 0.03$).

taneous evaporation of drops in a thin wall region. From equation (14) and Fig. 4 it is seen that the heat transfer enhancement can attain finite values even at a small concentration of drops. This effect is the stronger the smaller the temperature differences and the greater the vaporization heat of the drops.

5. INTENSIVE DEPOSITION OF DROPS

Consider the second limiting case, i.e. large drops when the Saffman force dominates in the interphase momentum exchange and the deposition of drops becomes appreciable. As follows from the results of calculations for the boundary layer with depositing particles [8], the involvement of the Saffman force leads to the disappearance of the effect of the accumulation of particles in a boundary layer. The time for which the depositing drops attain the wall is not large, therefore when the values of α and a are small, it is possible to neglect both the effect of the drops on the carrying phase movement and the variation in the radius of the drops until the time of their deposition. In this case, the surface immersed in flow is governed in the main by the evaporation of the drops falling out on the hot surface. Here the processes of the breaking of drops and the downstream sweeping of tiny evaporating drops in a thin wall layer are possible. At the present time, the theoretical description of these processes is difficult [13], therefore it will be assumed that a drop depositing on the surface absorbs the energy of evaporation. Then the total heat flux from the wall may be presented in the form

$$q^* = \left|\lambda^* \frac{\partial T^*}{\partial y^*} + m^* n^*_{sw} v^*_{sw} [H + c_s(T^*_w - T^*_{sw})]\right|.$$

The last term in square brackets may be omitted for the small temperature differences considered. In a non-dimensional form, the ratio of the heat flux q^* to its value in the case of the absence of drops q^*_0 has the form

$$\frac{q^*}{q^*_0} = \frac{Nu}{Nu_0} = 1 + \frac{\alpha H\, Pr^{2/3}}{0.332 c_p (T^*_w - T^*_\infty)} n_{sw} |v_{sw}| \sqrt{x}. \qquad (15)$$

For deriving equation (15), use was made of the expression for the heat flux from the plate in a pure gas [17]

$$q^*_0 = 0.332 \lambda^* (T^*_w - T^*_\infty) Pr^{1/3} \left(\frac{u^*_\infty}{v^* x^*}\right)^{1/2}.$$

As is seen from equation (15), in the approximation considered the study of the plate heat transfer enhancement was reduced to the determination of the mass flux of the drops depositing on the wall. This statement of the problem remains valid also for the case when the carrying phase is a mixture of the vapour of the substance of drops with an inert gas.

The constructed models of the gas-drop mixture flow in a boundary layer and relation (15) were used for comparison with experimental data [2] and explanation of the discovered effect of sharp heat transfer enhancement on a plate at a low concentration of drops. The authors of ref. [2] carried out an experimental investigation into the influence of the evaporation of water drops in an air boundary layer with longitudinal flow past a vertical plate with temperatures of 50 and 70°C. The external flow temperature was 20°C; the mean diameter of drops (calculated on the basis of the mean volume of drops) was equal to 6×10^{-3} cm. In experiments the concentration of drops did not exceed 1.67%.

The predicted integral heat transfer coefficient for a segment of the plate (2 cm $\leqslant x \leqslant$ 16 cm) will be compared with the experimental data of ref. [2]. Compute the quantity

$$A = \frac{1}{L}\int \frac{Nu}{Nu_0}\, dx^*, \quad 2\,\text{cm} \leqslant x^* \leqslant 16\,\text{cm}, \quad L = 14\,\text{cm}.$$

Equation (15) yields

$$A = 1 + \varphi \frac{l}{L} \int n_{sw} |v_{sw}| \sqrt{x}\, dx$$

$$\varphi = \frac{\alpha H\, Pr^{2/3}}{0.332 c_p (T^*_w - T^*_\infty)}. \qquad (16)$$

To calculate n_{sw} and v_{sw} the equations for a dispersed phase from system (8) were solved numerically at $a = 0$ and $\sigma = 1$. Neglecting the effect of drops, the carrying phase velocity field in the boundary layer is determined by solving the Blasius problem. It has the form [17]

$$u(x,\eta) = F', \quad v(x,\eta) = \frac{1}{2\sqrt{x}}\left(\frac{\eta F'}{\sqrt{x}} - F\right).$$

Here the function $F[\eta/\sqrt{x}]$ satisfies the boundary value problem

$$2F''' + FF'' = 0, \quad F(0) = F'(0) = 0, \quad F'(\infty) = 1.$$

The motion and continuity equations for the medium of the drops, considered on their fixed trajectory, may be reduced to a system of ordinary differential equations. For this purpose, the dimensionless time of the motion of drops along the trajectory $t = t^* u_\infty^*/l$ and the Lagrangian coordinate η_0 (the ordinate of the trajectory origin at $x = 0$) will be introduced as independent variables. With η_0 fixed, the particle medium motion equations and the boundary conditions will acquire the form

$$\frac{dx}{dt} = u_s, \quad \frac{d\eta}{dt} = v_s, \quad \frac{du_s}{dt} = (u - u_s)G$$

$$\frac{dv_s}{dt} = (v - v_s)G + \kappa(u - u_s)\bigg/\sqrt{\left(\frac{\partial u}{\partial \eta}\right)}$$

$$t = 0: \quad x = 0, \quad \eta = \eta_0, \quad u_s = 1, \quad v_s = 0. \quad (17)$$

The continuity equation in the selected Lagrangian coordinates has the form

$$\frac{1}{n_s(\eta_0 t)} = u_s g - v_s e, \quad e = \frac{\partial x(\eta_0, t)}{\partial \eta_0}, \quad g = \frac{\partial \eta(\eta_0, t)}{\partial \eta_0}. \quad (18)$$

To find n_s from equations (18) along the trajectory of drops, it is necessary to know the functions e and g. For their determination, differentiate equation (17) with respect to η_0. This will yield the equations and boundary conditions

$$\frac{de}{dt} = r, \quad \frac{dr}{dt} = c_1\left[1 + \frac{5}{18}R^{2/3}(u - u_s)^{2/3}\right], \quad \frac{dg}{dt} = w$$

$$\frac{dw}{dt} = c_2 G + \frac{R^{2/3}(v - v_s)c_1}{9(u - u_s)^{1/3}} + \kappa[c_1 + c_3(u - u_s)]\bigg/\sqrt{\left(\frac{\partial u}{\partial \eta}\right)}$$

$$c_1 = e\frac{\partial u}{\partial x} + g\frac{\partial u}{\partial \eta} - r, \quad c_2 = e\frac{\partial v}{\partial x} + g\frac{\partial v}{\partial \eta} - w$$

$$r = \frac{\partial u_s(\eta_0, t)}{\partial \eta_0}, \quad w = \frac{\partial v_s(\eta_0, t)}{\partial \eta_0}$$

$$c_3 = \left(e\frac{\partial^2 u}{\partial x\,\partial \eta} + g\frac{\partial^2 u}{\partial \eta^2}\right)\bigg/ 2\frac{\partial u}{\partial \eta}$$

$$t = 0: \quad e = 0, \quad r = 0, \quad g = 1, \quad w = 0. \quad (19)$$

For numerical calculations of equations (17)–(19) a new independent variable x was introduced, the resulting system of equations was integrated numerically by the Kutta–Merson method. The carrying phase velocity components and their derivatives in space were calculated with the use of the cubic interpolation of the tabulated values of the function F from ref. [17]. Figure 5 presents the comparison of the predicted values of A with the data obtained in processing the variables of experimental results of ref. [2]. The crosses, circles and triangles correspond to the experimental data for the external flow velocities of 980, 750 and 540 cm s^{-1}. The predicted results for the same velocities are shown by solid lines 1–3. In computations it was assumed that $\sigma = 3\times10^{-4}$ cm, $Pr = 0.72$, $c_p = 1$ J deg^{-1} g^{-1}, $H = 2.35\times10^3$ J g^{-1} deg^{-1}, $\rho_s^0/\rho^* = 10^3$, $\nu^* = 0.17$ cm^2 s^{-1}. Note that the Reynolds numbers for flow around drops in a boundary layer are not small under the considered conditions. This should be taken into account in the coefficient at the Saffman force. Based on the comparison, within a wide range of Reynolds numbers, between the predicted and experimental data on the trajectories of particles in a wall boundary layer behind a shock wave, a conclusion has been drawn [18] concerning the necessity for about a five-fold increase in the coefficient at the Saffman force at rather high Reynolds numbers. Therefore, calculations of the quantity A with the coefficient κ increased five-fold have been also conducted (dashed lines in Fig. 5). For $u_\infty^* = 540$ cm s^{-1} the dashed line coincides with solid line 2. It is seen that the dashed lines better fit the experimental data. The most important result was the determination of the governing parameter φ, equation (16), on which the degree of heat transfer enhancement depends. The finiteness of φ even at a very low concentration of drops explains the effect of heat transfer enhancement on a hot plate observed in experiments [2].

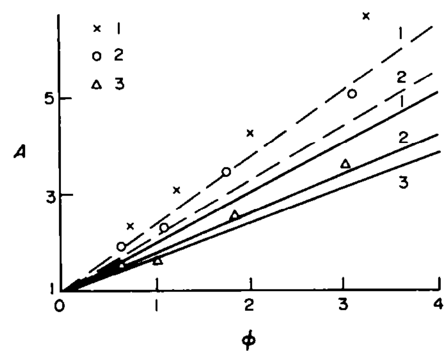

FIG. 5. Comparison of the predicted heat transfer coefficient for the section of the plate with the experimental data of ref. [2] (1, $u_\infty^* = 9.8$ m s^{-1}; 2, $u_\infty^* = 7.5$ m s^{-1}; 3, $u_\infty^* = 5.4$ m s^{-1}) the dashed lines indicate the calculation with the correction factor at the Saffman force.

In conclusion, it should be noted that currently the problem of a lift force acting on a sphere which moves in the shear velocity field of a gas has not been solved in rigorous formulation with allowance for finite inertia effects. A recent publication [19] does not take into account the origin of the angular rotation of a sphere. It should be also noted that the description of local heat transfer in the conditions of mist flow with the deposition of drops requires more detailed models of the interaction of drops with a hot surface, including the account for the fragmentation and evaporation of drop fractions in a wall layer.

6. CONCLUSION

A consistent asymptotic model of a laminar boundary layer in a two-phase 'gas-evaporating drops' medium is suggested with account for the inertia of drops. The problem of a two-phase boundary layer on a heated semi-infinite plate has been solved by numerical and asymptotic methods. It is shown that in the case of fine drops a region with a pure gas may appear within the boundary layer. At the boundary of this region the accumulation of drops occurs. Larger drops are carried out onto the wall due to the Saffman force acting on the sphere in a locally shear flow and their contact with the hot surface intensifies the heat transfer. Similarity parameters have been identified that characterize the degree of heat transfer enhancement by drops evaporating in the boundary layer. The increase in the heat transfer coefficient is shown to be proportional to the quantity $\alpha H/c_p(T^*_w - T^*_\infty)$. This effect can be significant even at small (about 1%) mass concentration of drops thus confirming the future prospects in using mist flows for enhancing heat transfer processes.

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